

OBJECTIVE In the year ahead, we will be investigating, discussing, and applying the introductory topics in the field of calculus. The focus of the course will be to master the concepts and techniques that will prepare you to continue to study calculus at a post-secondary institution. **This is a university course and will be taught as such.** At the end of the year, successful students will be able to challenge the AP exam, giving them the opportunity to receive credit for introductory calculus courses at post-secondary institutions world wide.

OUTLINE The following is an approximate order and time frame for what we will be studying:

UNIT 1 – Functions & Limits	19 classes
UNIT 2 – The Derivative	17 classes
UNIT 3 – Applications of the Derivative	15 classes
UNIT 4 – Inverse, Exponential, & Trigonometric Functions	15 classes
UNIT 5 – Integration	16 classes
UNIT 6 – Applications of Integration	12 classes
AP EXAM REVIEW & PRACTICE	EXAM: Wednesday, May 4, 2011

Some of the activities and assignments that we will be involved in as a class are listed below:

- Lectures
- Group Work Investigations
- Quizzes & Exams
- Assignments
- Discovery Based Worksheets
- Calculator/Computer Activities

MATERIALS NEEDED

Each student is expected to bring the following items to **every class**:

- Textbook
- Approved Graphing Calculator
- Binder, Pen, Pencil, Eraser, Ruler
- Lined and Non-metric Graph Paper

COURSE EVALUATION

Your grade will be determined according to your marks on:

- Assignments (20% of school mark)
- Quizzes (10% of school mark)
- Unit Exams (70% of school mark)
- AP Calculus Exam (AB)

COURSE EXPECTATIONS

- **Technology:** Students will receive frequent instruction on the use of the graphing calculator (TI-83+) to analyze graphs and equations and as a means of interpreting and solving problems. As such, it is imperative that students bring their calculator to class every day. The use of computer programs (such as the graphing tool developed by *Alberta Learning*) will also be used in conjunction with various lessons involving *Active Board* technology.
- **Homework:** As this is a very demanding course with new material presented almost daily, students are encouraged to discuss and complete assignments together. Every second weekend a short take home assignment will be given which will contain AP exam level questions that will be marked according to the AP scoring rubric.
- **Testing:** Each unit will consist of one or two quizzes and a unit exam. The quizzes are yours to keep and full answer keys will be posted in the room for your reference. There will always be at least one full day of in-class review before the major unit exam. These exams will involve multiple choice and written response questions and in some cases, calculators will not be allowed during the writing of the exam.

If you are having any difficulties at any time throughout the semester, please attend the daily help sessions right away so we can clear things up before it gets too late!!!

AP CALCULUS: UNIT 1 – FUNCTIONS & LIMITS

LESSON 1/2 – Review: Functions

- *Handout: More Stuff You Need To Know About Functions*
- Review domain & range, continuity, function notation and composition, inverses, and writing linear functions in different ways
- Introduce even/odd/one-to-one functions, piecewise functions, and interval notation
- *Pgs. 7: #11, 15, 19, 31 & Pgs. 17: #5, 9, 17-27, 33, 41, 49, 59, 63*

LESSON 3 – Review: More Function Practice

- Introduce the extension of $f(x) = e^x$ and $g(x) = \ln x$ for exponential and logarithmic ideas
- *Pgs. 24: #1-9, 17, 37 & Pgs. 39: #1-11, 19, 39*

LESSON 4 – Review: Algebra

- Review and extend the ideas of solving systems, factoring, rationalizing the denominator, and stating restrictions for given expressions
- *Handout: Pure Math Review & Extensions*

LESSON 5/6 – Introduction to Limits

- Introduce the idea of a limit using simple graphs and tables...use the calculator as an aid to this process
- Define the limit, emphasizing wording and proper mathematical notation
- Discuss how if $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x)$, then $\lim_{x \rightarrow a} f(x)$ exists
- *Pgs. 62: #1-15, 31, 45, 47, 49, 51, 59, 61*

LESSON 7 – The Limit Laws

- Discuss the evaluation of limits for polynomial and rational/radical functions by means of substitution and algebraic manipulation
- *Handout: The Limit Laws*

LESSON 8 – The Limit Laws: Trigonometry

- Discuss the limit identities of $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ and $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x} = 0$...need to memorize these!
- Lots of examples to show how to use the trig identities to evaluate limits by algebraic manipulation
- *Handout: The Limit Laws: Trigonometry*

LESSON 9 – Limits Involving Infinity

- Discuss the properties of horizontal/vertical asymptotes
- Focus on using graphs and table to determine the limits
- *Pgs. 71: #1-11, 15-19, 23-31, 47, 49*

LESSON 10 – Limits Involving Infinity

- Discuss how to algebraically determine limits for various rational functions involving polynomials, absolute value, and radicals
- *Handout: Limits To Infinity...And Beyond!*

LESSON 11 – Continuity

- Define continuity and types of discontinuity and discuss how to determine if a function is continuous at a point or on a given interval
- Graphical interpretation of continuity/discontinuity
- The Intermediate Value Theorem for functions
- *Pgs. 80: #1-15, 21, 23, 35, 37, 41, 47*

LESSON 12/13 – Tangent Lines and Rates of Change

- Discuss how average rates of change relate to slopes of secant lines
- Discuss how limits relate to the slopes of tangent lines and instantaneous rates of change
- *Pgs. 87: #1-11, 17, 19, 23, 25, 29*

LESSON 14/15/16 – Quiz/Review/Unit Exam

AP CALCULUS: UNIT 2 – THE DERIVATIVE

LESSON 1/2 – The Definition of the Derivative

- Define the derivative as the limit of the instantaneous rate of change over increasingly small intervals
- Discuss the language and notation of the derivative
- Given $f(x)$ as a table or graph, discuss how to create the graph of $f'(x)$
- Discuss that a function is differentiable and continuous at any point with one unique tangent line
- Define both the derivative as the limit of the quotient and as the slope of a curve at a point
- *Pgs. 101: #3, 11, 13, 19, 21*

LESSON 3 – Differentiability

- Discuss concept of *local linearity*
- Discuss how the derivative from the left and right must be equal for differentiability at that point
- Discuss how differentiability exists at the endpoints of a closed interval
- Show how to determine numerical derivatives on the calculator, and sketch the derivative of any function without knowing its equation
- The Intermediate Value Theorem for derivatives
- *Pgs. 101: #3, 11, 13, 19, 21*

LESSON 4/5/6 – The Laws of Derivatives

- Focus on the power, product, and quotient rules
- Have students complete derivatives using limits before being allowed to use power, product, or quotient rules and use the calculator to discuss how the graphs of the function and the derivative are related
- Lots and lots of practice, emphasizing that simplification is not required, but often useful
- *Handouts: The Derivative Laws: Power Rule, Product/Quotient Rules*

LESSON 7 – Trig Derivatives

- Discuss proofs of sine and cosine and how they relate to the derivatives of the 6 basic trig ratios
- *Pgs. 140: #1-13, 21, 33*

LESSON 8/9/10 – The Chain Rule

- Discuss simplifying the function when possible before determining the derivative
- *Handouts: The Derivative Laws: The Chain Rule*

LESSON 11 – Implicit Differentiation

- Focus on the notation that $y' = \frac{dy}{dx}$
- Lots of examples including trig and complex functions
- *Pgs. 155: #1-19*

LESSON 12 – Derivatives of Logarithms

- *Handouts: Exponential & Logarithmic Derivatives*
- Discuss proofs using implicit differentiation...discuss why the power rule won't work
- *Pgs. 170: #1-45*

LESSON 13 – Inverse Trigonometric Functions

- *Handouts: Derivative Of Trig & Inverse Trig Functions*
- Discuss the domain and range of the inverse trig functions and the proofs using implicit differentiation
- *Pgs. 162: #1-18*

LESSON 14 – Higher Derivatives

- *Handouts: Higher Derivatives*

LESSON 15 – Tangent Lines

- A variety of tangent line problems and other questions to understand the rules of derivatives

LESSON 16/17/18 – Quiz/Review/Unit Exam

AP CALCULUS: UNIT 3 – APPLICATIONS OF THE DERIVATIVE

LESSON 1/2 – Extreme Values

- Discuss absolute and relative max/min values for any function
- Introduce the idea of critical numbers in finding absolute and relative extrema
- Discuss the Extreme Value Theorem
- *Pgs. 184: #1-17, 21, 25, 29, 35, 39, 43, 47*

LESSON 3 – Mean Value Theorem & Antiderivatives

- Discuss the Mean Value Theorem
- Introduce basic antiderivatives that only require one step...discuss using initial values to solve for the unknown constant
- *Pgs. 192: #15, 17, 19, 21, 25-33*

LESSON 4 – Increasing/Decreasing Functions

- Define monotonic functions
- Discuss the first derivative test for determining intervals of increase/decrease on a given interval
- Discuss how the intervals of increase/decrease help determine the location of relative extremum
- *Pgs. 192: #1-13, 37, 57*

LESSON 5/6 – Concavity

- Use *Graphing Tool* from AB Learning to illustrate the connection between the graphs of $f(x)$, $f'(x)$, and $f''(x)$
- Discuss the need to simplify the 1st and 2nd derivatives as much as possible to aide in the solving of these equations
- Discuss the use of the second derivative to find points of inflection and relative max/min values
- *Pgs. 204: #5, 9, 11, 21, 23, 31, 33, 47*

LESSON 7/8 – Curve Sketching

- Students should be able to graph any given function by finding
 - Domain and range & intercepts
 - Symmetry (even/odd functions, periodic functions)
 - Asymptotes (vertical, horizontal, and slant...this is a new idea here)
 - Increases/decreases
 - Maximum and minimum values & Concavity and points of inflection
- Ensure some time to check these things on the graphing calculator
- *Handouts: Graphing...Putting It All Together!*

LESSON 9/10 – Position/Velocity/Acceleration

- Define the difference between average speed and average displacement
- Discuss that velocity and acceleration have direction...same direction means speeding up, opposite direction means slowing down
- Discuss how to diagram change of direction on a “line” using time and position
- *Pgs. 129: #3, 13, 21, 23, 29 Pgs. 140: #23 Pgs. 163: #23 Pgs. 205: 37, 39, 41*

LESSON 11/12/13 – Quiz/Review/Unit Exam

AP CALCULUS: UNIT 4 – PROBLEM SOLVING & THE DERIVATIVE

LESSON 1/2 – Rates of Change

- Use the knowledge of derivatives to extend the knowledge from Unit 1
- Introduce the “delta” notation
- Heavy emphasis on applications and problem solving
- *Pgs. 64: #57 Pgs. 88: #23, 25 Pgs. 121: #33, 35 Pgs. 130: 5, 9, 11, 15, 31 Pgs. 148: #61*

LESSON 3/4/5 – Related Rates

- Lots and lots of examples...discuss at length how to set up the problem
- Emphasize the need to differentiate WRT time and to only use variables that have information provided about them
- *Day 1 – Pgs. 237: #1-9*
- *Day 2 – Pgs. 238: #11, 13, 15, 21, 31*
- *Day 3 – Pgs. 238: #*

LESSON 6/7/8 – Optimization & Max/Min Problems

- Using the first derivative to find absolute maximum or minimum values
- Students should be able to use ideas from lesson 1-4 to prove that a certain value is the max/min value

LESSON 9 – Differentials & Linear Approximation

- Introduce the idea and notation of the differential : $dy = f'(x)dx$... $\Delta y = f(x + \Delta x) - f(x)$
- Finding tangent lines to a curve to approximate the curve and its applications

OPTIONAL LESSON – L'Hospital's Rule

- Discuss how derivative simplifies finding the limits of the form $\frac{0}{0}$, $0 \times \infty$, and $\frac{\infty}{\infty}$

OPTIONAL LESSON – Newton's Method

- Discuss how to find the roots of an equation

LESSON 10/11/12 – Quiz/Review/Unit Exam

AP CALCULUS: UNIT 5 – INTEGRATION

LESSON 1/2 – Estimating Area

- Discuss how to find the area under a curve by using rectangles and trapezoids
- Discuss left, right, and midpoint rectangling and how to use the concavity and increase/decrease of the curve to determine if the estimate is high or low
- *Handouts: Approximating The Area Under A Curve*

LESSON 3 – Intro To Integration: Antiderivatives

- Discuss the basic antiderivatives of polynomial, logarithmic, and trigonometric functions
- Introduce integration as another way to represent the antiderivative of a function
- *Pgs. 312: #1-23*

LESSON 4 – Area & Antiderivatives

- Introduce the notation and language of the definite integral $\int_a^b f(x)dx = F(b) - F(a)$ as being the area under the graph of a positive function and the limit of Riemann sums
- Discuss the conditions of a function which make it integrable
- Discuss how to also use the calculator to determine the value of a definite integral
- *Pgs. 267: #13, 15, 19, 21, 23, 25, 27, 29, 35, 39*

LESSON 5 – The Definite Integral & Average Value

- *Handouts: The Laws Of The Definite Integral*
- Discuss that the average value of a function is $\frac{1}{b-a} \int_a^b f(x)dx$ and that the mean value theorem applies to integration as well as derivatives
- More practice using antiderivatives to evaluate integration
- *Pgs. 267: #1-27*

LESSON 6/7 – The Fundamental Theorem of Calculus

- *Handouts: The Fundamental Theorem of Calculus*

LESSON 8/9/10 – Integration by Substitution

- Simplification through substitution...discuss its relationship to the chain rule of differentiation
- Changing the limits of integration in a definite integral
- Discuss quicker methods of integrating functions symmetric about the y-axis and using trig identities
- *Handouts: Integration By Substitution*

LESSON 11/12/13 – Quiz/Review/Unit Exam

AP CALCULUS: UNIT 6 – APPLICATIONS OF INTEGRATION

LESSON 1/2 – Area Between Two Curves

- Discuss the need to know which is the greater function
- Review solutions both with and without using the calculator
- Discuss idea of expressing the functions as values of y to avoid working with more than one integral
- *Pgs. 380: #5, 7, 13, 3, 19, 27, 41...no calculator for question where the graph is given*

LESSON 3/4/5 – Volume

- *Day 1* – Volume by area of cross-section
- *Day 2* – Volume by revolution about the x/y -axis (disks only)
- *Day 3* – Volume by revolution (washers) and other horizontal/vertical lines
- *Shell method for more volumes of revolution involving unclear limits of integration (if time)

LESSON 6 – Position/Velocity/Acceleration

- Focus on question where the velocity or acceleration function is given
- Review the idea of *average value* and how it applies to these ideas
- Emphasize the difference between total *distance* and total *displacement*
- *Handout: More Position, Velocity, & Acceleration*

LESSON 7/8 – Differential Equations

- Solving by integration...it is important that students really understand the meaning of $y' = \frac{dy}{dx}$
- Solving initial value problems
- Solving separable differential equations
- *Handout: Pgs. 35-38, 44 The Teaching Series: Differential Equations (#1, 2, 5, 7, 9, 11, 13, 2001, 2000)*

LESSON 9 – Exponential Growth & Decay

- Discuss that if a “population” changes at a rate proportional to the present amount then $y' = ky$ is the differential equation whose initial value solution is $y = y_0 e^{kt}$
- *Handout: Pgs. 35-38, 44 The Teaching Series: Differential Equations (# 4, 6, 12, 1993)*
Textbook Pgs. 339: #27, 29

LESSON 10 – Slope Fields

- Interpreting differential equation using slope fields and relating slope fields to the solution curves of differential equations
- *Handout: Slope Fields*

LESSON 11 – Rates of Change Revisited

- Focus on questions where the rate of change function is known and integration is needed to calculate a total result
- *Handout: More Rates of Change*

LESSON 12/13/14 – Quiz/Review/Unit Exam